Density Fluctuations as Signature of a Non–Equilibrium First Order Phase Transition

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Abstract.
We show that in the presence of spinodal instabilities which develop at a first order phase transition, the fluctuations of conserved charges can be as strong as those at the critical end point (CEP). In particular, the net baryon number susceptibility diverges as the system crosses the isothermal spinodal lines. This indicates that charge density fluctuations can be used not only to probe the CEP but also the non–equilibrium first order chiral phase transition in heavy ion collisions.

PACS numbers: 25.75.Nq, 24.60.Lz

1. Introduction

The search for the critical end point (CEP) [1] has attracted a considerable attention in heavy-ion phenomenology [2]. It is of particular interest to identify its position in the phase diagram and to study general properties of thermodynamic quantities in its vicinity. Modifications in the magnitude of fluctuations or the corresponding susceptibilities can be considered as a possible signal for deconfinement and chiral symmetry restoration [3, 2, 4]. In this context, fluctuations related to conserved charges play an important role since they are directly accessible in experiments [5, 6].

The enhancement of baryon number fluctuations could be a clear indication for the existence of the CEP in the QCD phase diagram. However, the suppression of density fluctuations along the first-order transition appear under the assumption that this transition takes place in equilibrium. This is modified when there is a deviation from equilibrium [7]. In this contribution we briefly show that enhanced baryon number density fluctuations is a signal for the first-order phase transition in the presence of spinodal decomposition.

2. The role of spinodal instabilities in fluctuations

In a non-equilibrium system, a first-order phase transition is intimately linked with the existence of a convex anomaly in the thermodynamic pressure [8]. There is an interval of energy density or baryon number density where the derivative of the pressure, $\partial P/\partial V > 0$, is positive. This anomalous behavior characterizes a region of instability in the (T, n_q)-plane which is bounded by the spinodal lines, where the pressure derivative with respect to volume vanishes. The derivative taken at constant temperature and that taken at constant entropy,

$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \qquad \text{and} \qquad \left(\frac{\partial P}{\partial V}\right)_S = 0\,, \tag{1}$$

define the isothermal and isentropic spinodal lines respectively.

If the first-order phase transition takes place in equilibrium, there is a coexistence region, which ends at the CEP. However, in a non-equilibrium first-order phase transition, the system supercools/superheats and, if driven sufficiently far from equilibrium, it becomes locally unstable due to the convex anomaly. In other words, in the coexistence region there is a range of densities and temperatures, bounded by the spinodal lines, where the spatially uniform system is mechanically unstable. Spinodal decomposition is thought to play a dominant role in the dynamics of low energy nuclear collisions in the regime of the first-order nuclear liquid-gas transition [8, 9]. In connection with the chiral and deconfinement phase transitions, the possibility of spinodal decomposition in heavy ion collisions has been discussed in [8, 9, 10].

In Fig. 1-left we show the evolution of the net quark number fluctuations along a path of fixed $T = 50$ MeV calculated in the Nambu–Jona-Lasinio (NJL) model in the mean field approximation [11]. When entering the coexistence region, a singularity in χ_q appears at the isothermal spinodal lines, where the fluctuations diverge and the susceptibility changes sign. In between the spinodal lines, the susceptibility is negative. This implies instabilities in the baryon number fluctuations when crossing from a meta-stable to an unstable phase. The above behavior of χ_q is a direct consequence of the thermodynamics relation

$$\left(\frac{\partial P}{\partial V}\right)_T = -\frac{n_q^2}{V}\frac{1}{\chi_q}\,. \tag{2}$$

Along the isothermal spinodals the pressure derivative in Eq. (2) vanishes. Thus, for non-vanishing density n_q, χ_q must diverge to satisfy (2). Furthermore, since the pressure derivative $\partial P/\partial V|_T$ changes sign when crossing the spinodal line, there must be a corresponding sign change in χ_q, as seen in Fig. 1-left.

In Fig. 1-right we show the evolution of the singularity at the spinodal lines in the T-n_q plane. The critical exponent at the isothermal spinodal line is found to be $\gamma = 1/2$, with $\chi_q \sim (\mu - \mu_c)^{-\gamma}$, while $\gamma = 2/3$ at the CEP [7]. Thus, the singularities at the two spinodal lines conspire to yield a somewhat stronger divergence as they join at the CEP. The critical region of enhanced susceptibility around the CEP is fairly small [12, 13],

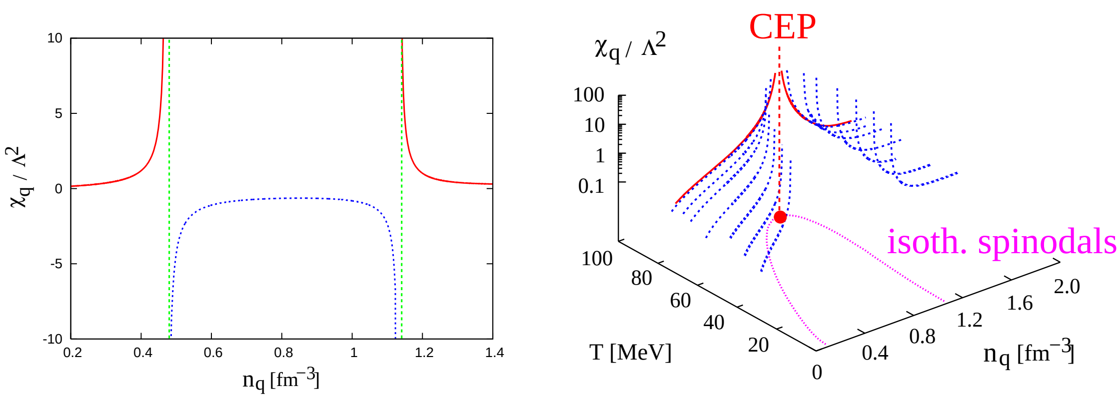

Figure 1. (Left) The net quark number susceptibility at $T = 50$ MeV as a function of the quark number density across the first-order phase transition. (Right) The net quark number susceptibility in the stable and meta-stable regions.

while in the more realistic non-equilibrium system one expects fluctuations in a larger region of the phase diagram, i.e., over a broader range of beam energies, due to the spinodal instabilities.

The rate of change in entropy with respect to temperature at constant pressure gives the specific heat expressed as

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = TV\left[\chi_{TT} - \frac{2s}{n_q}\chi_{\mu T} + \left(\frac{s}{n_q}\right)^2 \chi_q\right]. \tag{3}$$

The entropy χ_{TT} and mixed $\chi_{\mu T}$ susceptibilities exhibit the same critical behaviors as that of χ_q shown in Fig. 1-left. Thus C_P also diverges on the isothermal spinodal lines and becomes negative in the mixed phase ‡. It was argued that in low energy nuclear collisions the negative specific heat could be a signal of the liquid-gas phase transition [14]. Its occurrence has recently been reported as the first experimental evidence for such an anomalous behavior [15].

3. Conclusions

We showed that in the presence of spinodal instabilities the net quark number fluctuations diverge at the isothermal spinodal lines of the first-order chiral phase transition. As the system crosses this line, it becomes unstable with respect to spinodal decomposition. The unstable region is in principle reachable in non-equilibrium systems that is most likely created in heavy ion collisions. Consequently, large fluctuations of baryon and electric charge densities are expected to probe not only the CEP but also a first order phase transition when the system crosses the spinodal lines.

‡ The specific heat with constant volume, on the other hand, continuously changes with n_q and has no singularities on the mean-field level.

Acknowledgments

C.S. acknowledges partial support by DFG cluster of excellence "Origin and Structure of the Universe". K.R. acknowledges partial support of the Polish Ministry of National Education (MENiSW) and DFG under the "Mercator program".

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